Name Key Date________ Class_____

Section 4.9 – Antiderivatives Worksheet
I.S. Calculus

For #1-4, match each function with its antiderivative.

1. $f(x) = \sin x$

B

A. $F(x) = \cos(1-x)$

$f(x) = -\sin(1-x)(-1)$

2. $f(x) = x\sin(x^2)$

C

B. $F(x) = -\cos x$

$f(x) = -(-\sin x)$

3. $f(x) = \sin(1-x)$

A

C. $F(x) = -\frac{1}{2}\cos(x^2)$

$f(x) = -\frac{1}{2}(-\sin x^2)(2x)$

4. $f(x) = x\sin x$

D

D. $F(x) = \sin x - x\cos x$

$f(x) = \cos x - x(-\sin x) + (\cos x)(-1)$

Evaluate each indefinite integral.

5. $\int (9x+2)\,dx$

$\frac{9x^2}{2} + 2x + C$

6. $\int \frac{dx}{x^3} = \int x^{-3}\,dx$

$= \frac{x^{-2}}{-2} + C = \frac{-1}{2x^2} + C$

7. $\int\left(\frac{t^3}{4}-\frac{1}{4t^3}\right)dt = \int\left(\frac{1}{4}t^3-\frac{1}{4}t^{-3}\right)dt$

$= \frac{1}{4}\cdot\frac{1}{4}t^4 - \frac{1}{4}\cdot\frac{t^{-2}}{-2} + C$

$= \frac{t^4}{16} + \frac{1}{8t^2} + C$

8. $\int x(x^2-4)dx = \int(x^3-4x)dx$

$= \frac{1}{4}x^4 - \frac{4}{2}x^2 + C$

$= \boxed{\frac{x^4}{4} - 2x^2 + C}$

9. $\int\sqrt{x}(5x^{-2}-2)dx$

$= \int(5x^{-\frac{3}{2}} - 2x^{\frac{1}{2}})dx$

$= 5\cdot 2x^{-\frac{1}{2}} - 2\cdot\frac{2}{3}x^{\frac{3}{2}} + C$

$= \frac{-10}{\sqrt{x}} - \frac{4x^{\frac{3}{2}}}{3} + C$

10. $\int\frac{x^3-4}{x^2}dx = \int(x-4x^{-2})dx$

$= \frac{1}{2}x^2 - 4\cdot\frac{x^{-1}}{-1} + C$

$= \frac{1}{2}x^2 + \frac{4}{x} + C$

11. $\int\frac{12-z}{\sqrt{z}}dz = \int\left(12z^{-\frac{1}{2}} - z^{\frac{1}{2}}\right)dz$

$= 12\cdot 2z^{\frac{1}{2}} - \frac{2}{3}z^{\frac{3}{2}} + C$

$= \boxed{24z^{\frac{1}{2}} - \frac{2}{3}z^{\frac{3}{2}} + C}$

12. $\int 12\sec x\tan x\,dx$

$= 12\sec x + C$

13. $\int(\theta+\sec^2\theta)d\theta$

$= \frac{1}{2}\theta^2 + \tan\theta + C$

14. $\int\left(\frac{3}{4x^2}+\csc^2 x\right)dx$

$= \int\left(\frac{3}{4}x^{-2} + \csc^2 x\right)dx$

$= \frac{3}{4}\cdot\frac{x^{-1}}{-1} - \cot x + C$

$= \boxed{\frac{-3}{4x} - \cot x + C}$

Solve each initial value problem.

15. $\frac{dy}{dt} = 2t + 9t^2 \qquad y(1) = 2$

$y = t^2 + \frac{9}{3}t^3 + C$

$2 = 1 + 3 + C$

$C = -2$

$\boxed{y = t^2 + 3t^3 - 2}$

16. $\frac{dy}{dx} = 8x^3 + 3x^2 \qquad y(2) = 0$

$y = \frac{8}{4}x^4 + \frac{3}{3}x^3 + C$

$0 = 2(2)^4 + (2)^3 + C$

$C = -40$

$\boxed{y = 2x^4 + x^3 - 40}$

17. $\frac{dz}{dt} = t^{-\frac{3}{2}} \qquad z(4) = -1$

$z = -2t^{-\frac{1}{2}} + C$

$-1 = \frac{-2}{\sqrt{4}} + C$

$-1 = -1 + C$

$C = 0$

$\boxed{z = \frac{-2}{\sqrt{t}}}$

18. $\frac{dy}{dx} = \frac{1}{3x^2} - \frac{x^2}{3} \qquad y(-6) = -3$

$\frac{dy}{dx} = \frac{1}{3}x^{-2} - \frac{1}{3}x^2$

$y = \frac{1}{3} \cdot \frac{x^{-1}}{-1} - \frac{1}{3} \cdot \frac{x^3}{3} + C$

$-3 = -\frac{1}{3(-6)} - \frac{(-6)^3}{9} + C$

$C = \frac{-487}{18} \qquad y = \frac{-1}{3x} - \frac{x^3}{9} - \frac{487}{18}$

19. $\frac{dy}{dx} = \sin x \qquad y\left(\frac{\pi}{2}\right) = 1$

$y = -\cos x + C$

$1 = -\cos\frac{\pi}{2} + C$

$C = 1$

$\boxed{y = -\cos x + 1}$

20. $\frac{dy}{dx} = \csc x \cot x \qquad y\left(\frac{11\pi}{6}\right) = 3$

$y = -\csc x + C$

$3 = -\csc\left(\frac{11\pi}{6}\right) + C$

$3 = -(2) + C$

$C = 1$

$\boxed{y = -\csc x + 1}$

Find the function f(x) that satisfies the given conditions.

21. $f''(x) = x^3 - 2x + 1$ $f'(1) = 0$ $f(0) = 4$

$f'(x) = \frac{1}{4}x^4 - x^2 + x + C_1$

$0 = \frac{1}{4} - 1 + 1 + C_1$

$C_1 = -\frac{1}{4}$

$f'(x) = \frac{1}{4}x^4 - x^2 + x - \frac{1}{4}$

$f(x) = \frac{1}{20}x^5 - \frac{1}{3}x^3 + \frac{1}{2}x^2 - \frac{1}{4}x + C_2$

$4 = C_2$

$\boxed{f(x) = \frac{1}{20}x^5 - \frac{1}{3}x^3 + \frac{1}{2}x^2 - \frac{1}{4}x + C_2}$

22. $f''(x) = \frac{2}{3x^{\frac{3}{2}}}$ $f'(4) = 1$ $f(9) = \frac{3}{2}$

$f''(x) = \frac{2}{3}x^{-\frac{3}{2}}$

$f'(x) = \frac{2}{3} \cdot -2x^{-\frac{1}{2}} + C_1$

$1 = \frac{-4}{3\sqrt{4}} + C_1$ $C_1 = \frac{5}{3}$

$f'(x) = \frac{-4}{3}x^{-\frac{1}{2}} + \frac{5}{3}$

$f(x) = \frac{-4}{3}2x^{\frac{1}{2}} + \frac{5}{3}x + C_2$

$\frac{3}{2} = \frac{-8}{3}\sqrt{9} + \frac{5}{3}(9) + C_2$ $\boxed{f(x) = \frac{-8}{3}\sqrt{x} + \frac{5}{3}x - \frac{11}{2}}$

$C_2 = -\frac{11}{2}$

23. $f''(x) = x - \cos x$ $f'(0) = 2$ $f(0) = -2$

$f'(x) = \frac{1}{2}x^2 - \sin x + C_1$

$2 = C_1$

$f'(x) = \frac{1}{2}x^2 - \sin x + 2$

$f(x) = \frac{1}{2} \cdot \frac{1}{3}x^3 + \cos x + 2x + C_2$

$-2 = 1 + C_2$

$C_2 = -3$

$\boxed{f(x) = \frac{1}{6}x^3 + \cos x + 2x - 3}$

24. $f''(x) = \frac{1 + x^2}{\sqrt{x}}$ $f'(1) = 3$ $f(0) = 2$

$f''(x) = x^{-\frac{1}{2}} + x^{\frac{3}{2}}$

$f'(x) = 2x^{\frac{1}{2}} + \frac{2}{3}x^{\frac{5}{2}} + C_1$

$3 = 2 + \frac{2}{3} + C_1$

$C_1 = \frac{1}{3}$

$f'(x) = 2x^{\frac{1}{2}} + \frac{2}{3}x^{\frac{5}{2}} + \frac{1}{3}$

$f(x) = 2\,\frac{2}{3}x^{\frac{3}{2}} + \frac{2}{3} \cdot \frac{2}{7}x^{\frac{7}{2}} + \frac{1}{3}x + C_2$

$2 = C_2$

$\boxed{f(x) = \frac{4}{3}x^{\frac{3}{2}} + \frac{4}{21}x^{\frac{7}{2}} + \frac{1}{3}x + 2}$